

Resolving the octant of θ_{23} via radiative μ - τ symmetry breaking

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Abstract

We point out that the observed neutrino mixing pattern at low energies is very likely to originate from the 3×3 Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton flavor mixing matrix U which possesses the exact μ - τ permutation symmetry $|U_{\mu i}| = |U_{\tau i}|$ (for $i = 1, 2, 3$) at a superhigh energy scale $\Lambda_{\mu\tau} \sim 10^{14}$ GeV. The deviation of θ_{23} from 45° and that of δ from 270° in the standard parametrization of U are therefore a natural consequence of small PMNS μ - τ symmetry breaking via the renormalization-group equations (RGEs) running from $\Lambda_{\mu\tau}$ down to the electroweak scale $\Lambda_{EW} \sim 10^2$ GeV. In fitting current experimental data we find that the RGE-corrected value of θ_{23} is uniquely correlated with the neutrino mass ordering: $\theta_{23} \simeq 42.4^\circ$ reported by Capozzi *et al* (or $\theta_{23} \simeq 48.9^\circ$ reported by Forero *et al*) at Λ_{EW} can arise from $\theta_{23} = 45^\circ$ at $\Lambda_{\mu\tau}$ in the minimal supersymmetric standard model if the neutrino mass ordering is inverted (or normal). Accordingly, the preliminary best-fit results of δ at Λ_{EW} can also evolve from $\delta = 270^\circ$ at $\Lambda_{\mu\tau}$ no matter whether the massive neutrinos are Dirac or Majorana particles.

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1 Introduction

From the discovery of atmospheric neutrino oscillations in 1998 [1] until the observation of the smallest neutrino mixing angle θ_{13} in 2012 [2], experimental neutrino physics was in full flourish. Today the era of precision measurements has come. A number of undergoing and upcoming neutrino oscillation experiments aim to determine the neutrino mass ordering, to probe the octant of the largest neutrino mixing angle θ_{23} , and to measure the Dirac CP-violating phase δ . Such knowledge will be fundamentally important, as it can help identify the underlying flavor symmetry or dynamics behind the observed pattern of lepton flavor mixing.

As far as the octant of θ_{23} is concerned, it is desirable to know whether this “atmospheric neutrino mixing” angle deviates from 45° or not, and if it does, how large or small the deviation is and in what direction the deviation evolves. A global analysis of current neutrino oscillation data done by Capozzi *et al* [3] yields the best-fit result $\theta_{23} \simeq 41.4^\circ$ (normal neutrino mass ordering) or $\theta_{23} \simeq 42.4^\circ$ (inverted neutrino mass ordering), which has a preference for the first octant (i.e., $\theta_{23} < 45^\circ$). In contrast, another best-fit result reported by Forero *et al* [4] is $\theta_{23} \simeq 48.8^\circ$ (normal ordering) or $\theta_{23} \simeq 49.2^\circ$ (inverted ordering), by which the second octant (i.e., $\theta_{23} > 45^\circ$) is favored. In both cases $\theta_{23} = 45^\circ$ will be allowed when the 1σ or 2σ error bars are taken into account. Hence the octant of θ_{23} remains an open issue, and a resolution to this puzzle awaits more accurate experimental data.

On the other hand, the best-fit results of δ in both Ref. [3] and Ref. [4] are close to an especially interesting value, 270° , although the confidence level remains quite low. In fact, the T2K measurement of a relatively strong $\nu_\mu \rightarrow \nu_e$ appearance signal [5] plays a crucial role in the global fit to make θ_{13} consistent with the Daya Bay result [2] and drive a slight but intriguing preference for $\delta \simeq 270^\circ$ [3, 4]. If this preliminary expectation turns out to be true, there will be no problem to observe significant effects of leptonic CP violation in the forthcoming long-baseline neutrino oscillation experiments.

On the theoretical side, $\theta_{23} = 45^\circ$ and $\delta = 270^\circ$ are a straightforward consequence of the μ - τ permutation symmetry manifesting itself in the 3×3 Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton flavor mixing matrix U [6]: $|U_{\mu i}| = |U_{\tau i}|$ (for $i = 1, 2, 3$), which can easily be embedded in an explicit flavor symmetry model. Hence the deviation of θ_{23} from 45° and that of δ from 270° must be related to small PMNS μ - τ symmetry breaking effects. This observation is important and suggestive, implying that the observed pattern of the PMNS matrix U should have an approximate μ - τ symmetry of the form $|U_{\mu i}| \simeq |U_{\tau i}|$ at low energies [7]. In comparison, the Cabibbo-Kobayashi-Maskawa (CKM) quark flavor mixing matrix V [8] does not possess such a peculiar structure.

In this work we pay particular attention to a very real possibility: the PMNS μ - τ symmetry is exact at a superhigh energy scale $\Lambda_{\mu\tau}$ where both tiny neutrino masses and large neutrino mixing angles could naturally be explained in a well-founded theoretical framework (e.g., with the canonical seesaw mechanism [9] and proper flavor symmetry groups [10]). In this case we find that it is possible to resolve the octant of θ_{23} and the quadrant of δ via radiative μ - τ symmetry breaking effects. Namely, the equalities $|U_{\mu i}| = |U_{\tau i}|$ are more or less violated when they evolve from $\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to the electroweak scale $\Lambda_{\text{EW}} \sim 10^2$ GeV via the relevant renormalization-group equations (RGEs), such that the correct octant of θ_{23} and the correct quadrant of δ can consequently be obtained. We carry out a numerical analysis of the issue for both Dirac and Majorana neutrinos based on the one-loop RGEs in the minimal supersymmetric standard model

(MSSM)¹. A striking finding of ours in fitting current neutrino oscillation data is that the RGE-corrected value of θ_{23} is uniquely correlated with the neutrino mass ordering: $\theta_{23} \simeq 42.4^\circ$ reported in Ref. [3] (or $\theta_{23} \simeq 48.9^\circ$ reported in Ref. [4]) at Λ_{EW} can evolve from $\theta_{23} = 45^\circ$ at $\Lambda_{\mu\tau}$ only when the neutrino masses have an inverted (or normal) ordering. Accordingly, the preliminary best-fit results of δ at Λ_{EW} can also originate from $\delta = 270^\circ$ at $\Lambda_{\mu\tau}$ thanks to radiative μ - τ symmetry breaking. Such remarkable results are independent of any specific models of neutrino mass generation and lepton flavor mixing, and they will soon be tested in the upcoming precision experiments of neutrino oscillations.

2 The RGEs of μ - τ symmetry breaking

The PMNS lepton flavor mixing matrix can be parametrized in the following “standard” way [12]:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu, \quad (1)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$ (for $ij = 12, 13, 23$), δ is referred to as the Dirac CP-violating phase, and $P_\nu = \text{Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ contains two extra phase parameters if massive neutrinos are the Majorana particles. Up to now θ_{12} , θ_{13} and θ_{23} have all been measured to a good degree of accuracy, and some preliminary hints for a nontrivial value of δ have also been obtained from a global analysis of current neutrino oscillation data [3, 4]. Here we are concerned about the three PMNS μ - τ “asymmetries”:

$$\begin{aligned} \Delta_1 &\equiv |U_{\tau 1}|^2 - |U_{\mu 1}|^2 = (\cos^2 \theta_{12} \sin^2 \theta_{13} - \sin^2 \theta_{12}) \cos 2\theta_{23} - \sin 2\theta_{12} \sin \theta_{13} \sin 2\theta_{23} \cos \delta, \\ \Delta_2 &\equiv |U_{\tau 2}|^2 - |U_{\mu 2}|^2 = (\sin^2 \theta_{12} \sin^2 \theta_{13} - \cos^2 \theta_{12}) \cos 2\theta_{23} + \sin 2\theta_{12} \sin \theta_{13} \sin 2\theta_{23} \cos \delta, \\ \Delta_3 &\equiv |U_{\tau 3}|^2 - |U_{\mu 3}|^2 = \cos^2 \theta_{13} \cos 2\theta_{23}, \end{aligned} \quad (2)$$

which satisfy the sum rule $\Delta_1 + \Delta_2 + \Delta_3 = 0$. All the three Δ_i vanish when the exact μ - τ permutation symmetry holds.

We conjecture that the exact PMNS μ - τ symmetry (i.e., $\Delta_i = 0$) can be realized at $\Lambda_{\mu\tau} \sim 10^{14}$ GeV in a given neutrino mass model with a proper flavor symmetry group [10]. In view of the facts that a nonzero and relatively large θ_{13} has been observed and the preliminary best-fit value of δ is not far from 270° at the electroweak scale [3, 4], we infer that the condition for all the three Δ_i to vanish should naturally be $\theta_{23} = 45^\circ$ and $\delta = 270^\circ$ at the μ - τ symmetry scale². In this case $\Delta_i \neq 0$ can therefore be achieved at $\Lambda_{\text{EW}} \sim 10^2$ GeV through the RGE running effects. The one-loop RGEs of $|U_{\alpha i}|^2$ (for $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$) have been derived by one of us in Ref. [14]. So it is straightforward to write out the RGEs of Δ_i in the MSSM as follows.

¹In this connection the standard-model RGEs are less interesting for two reasons: (a) it will be difficult to make the deviation of θ_{23} from 45° appreciable even if the neutrino masses are nearly degenerate; (b) the standard model itself will largely suffer from the vacuum-stability problem for the measured value of the Higgs mass ($\simeq 125$ GeV) as the energy scale is above 10^{10} GeV [11].

²Although $\Delta_i = 0$ might also result from $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$ or $\theta_{23} = 45^\circ$ and $\delta = 90^\circ$ [13], neither of them is close to the best-fit results of the lepton flavor mixing parameters reported in Refs. [3] and [4]. These two possibilities are much less likely because they have to invoke violent RGE running effects between $\Lambda_{\mu\tau}$ and Λ_{EW} in order to fit the present experimental data, which actually favor slight μ - τ symmetry breaking [7]. That is why we concentrate our interest only on the possibility of $\theta_{23} = 45^\circ$ and $\delta = 270^\circ$ in this paper.

2.1 Dirac neutrinos

If massive neutrinos are the Dirac particles, we find

$$\begin{aligned}
16\pi^2 \frac{d\Delta_1}{dt} &= -y_\tau^2 [\xi_{21} (|U_{\tau 1}|^2 \Delta_2 + |U_{\tau 2}|^2 \Delta_1 + |U_{e 3}|^2) + \xi_{31} (|U_{\tau 1}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_1 + |U_{e 2}|^2)] , \\
16\pi^2 \frac{d\Delta_2}{dt} &= +y_\tau^2 [\xi_{21} (|U_{\tau 1}|^2 \Delta_2 + |U_{\tau 2}|^2 \Delta_1 + |U_{e 3}|^2) - \xi_{32} (|U_{\tau 2}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_2 + |U_{e 1}|^2)] , \\
16\pi^2 \frac{d\Delta_3}{dt} &= +y_\tau^2 [\xi_{31} (|U_{\tau 1}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_1 + |U_{e 2}|^2) + \xi_{32} (|U_{\tau 2}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_2 + |U_{e 1}|^2)] , \quad (3)
\end{aligned}$$

where $t \equiv \ln(\mu/\Lambda_{\mu\tau})$ with μ being an arbitrary scale between Λ_{EW} and $\Lambda_{\mu\tau}$, $y_\tau^2 = (1 + \tan^2 \beta) m_\tau^2/v^2$ is the Yukawa coupling eigenvalue of the tau lepton in the MSSM with $\tan \beta$ and v being self-explaining, and $\xi_{ij} \equiv (m_i^2 + m_j^2)/\Delta m_{ij}^2$ with $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ being the neutrino mass-squared differences. Given the fact $|\Delta m_{31}^2| \simeq |\Delta m_{32}^2| \sim 30\Delta m_{21}^2$ with $\Delta m_{21}^2 \simeq 7.5 \times 10^{-5} \text{ eV}^2$, $\xi_{21} \gg |\xi_{31}| \simeq |\xi_{32}|$ is expected to hold in most cases. But this does not necessarily mean that the μ - τ asymmetry Δ_3 should be more stable against radiative corrections than the other two asymmetries. The reason is simply that the running behaviors of Δ_i depend also on the initial inputs of all the nine $|U_{\alpha i}|^2$. In general, however, an appreciable deviation of θ_{23} from 45° (i.e., an appreciable deviation of Δ_3 from zero) requires a sufficiently large value of $\tan \beta$, and its evolving direction is governed by the neutrino mass ordering or equivalently the sign of Δm_{31}^2 or Δm_{32}^2 .

2.2 Majorana neutrinos

If massive neutrinos are the Majorana particles, we arrive at

$$\begin{aligned}
16\pi^2 \frac{d\Delta_1}{dt} &= -y_\tau^2 \{ \xi_{21} (|U_{\tau 1}|^2 \Delta_2 + |U_{\tau 2}|^2 \Delta_1 + |U_{e 3}|^2) + \xi_{31} (|U_{\tau 1}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_1 + |U_{e 2}|^2) \\
&\quad + \zeta_{21} [(|U_{\tau 1}|^2 \Delta_2 + |U_{\tau 2}|^2 \Delta_1 + |U_{e 3}|^2) \cos \Phi_{12} + \mathcal{J} \sin \Phi_{12}] \\
&\quad + \zeta_{31} [(|U_{\tau 1}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_1 + |U_{e 2}|^2) \cos \Phi_{13} - \mathcal{J} \sin \Phi_{13}] \} , \\
16\pi^2 \frac{d\Delta_2}{dt} &= +y_\tau^2 \{ \xi_{21} (|U_{\tau 1}|^2 \Delta_2 + |U_{\tau 2}|^2 \Delta_1 + |U_{e 3}|^2) - \xi_{32} (|U_{\tau 2}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_2 + |U_{e 1}|^2) \\
&\quad + \zeta_{21} [(|U_{\tau 1}|^2 \Delta_2 + |U_{\tau 2}|^2 \Delta_1 + |U_{e 3}|^2) \cos \Phi_{12} + \mathcal{J} \sin \Phi_{12}] \\
&\quad - \zeta_{32} [(|U_{\tau 2}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_2 + |U_{e 1}|^2) \cos \Phi_{23} + \mathcal{J} \sin \Phi_{23}] \} , \\
16\pi^2 \frac{d\Delta_3}{dt} &= +y_\tau^2 \{ \xi_{31} (|U_{\tau 1}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_1 + |U_{e 2}|^2) + \xi_{32} (|U_{\tau 2}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_2 + |U_{e 1}|^2) \\
&\quad + \zeta_{31} [(|U_{\tau 1}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_1 + |U_{e 2}|^2) \cos \Phi_{13} - \mathcal{J} \sin \Phi_{13}] \\
&\quad + \zeta_{32} [(|U_{\tau 2}|^2 \Delta_3 + |U_{\tau 3}|^2 \Delta_2 + |U_{e 1}|^2) \cos \Phi_{23} + \mathcal{J} \sin \Phi_{23}] \} , \quad (4)
\end{aligned}$$

where $\zeta_{ij} \equiv 2m_i m_j / \Delta m_{ij}^2$, $\cos \Phi_{ij} \equiv \text{Re}(U_{\tau i} U_{\tau j}^*)^2 / |U_{\tau i} U_{\tau j}^*|^2$, $\sin \Phi_{ij} \equiv \text{Im}(U_{\tau i} U_{\tau j}^*)^2 / |U_{\tau i} U_{\tau j}^*|^2$, and the leptonic Jarlskog invariant \mathcal{J} [15] is defined through

$$\text{Im}(U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\beta i}^*) = \mathcal{J} \sum_{\gamma} \epsilon_{\alpha\beta\gamma} \sum_k \epsilon_{ijk} \quad (5)$$

with the Greek and Latin subscripts running over (e, μ, τ) and $(1, 2, 3)$, respectively. Since the sign of ζ_{ij} is always the same as that of ξ_{ij} , it is possible to adjust the evolving direction of Δ_3 without much fine-tuning of the other relevant parameters. Hence, similar to the Dirac neutrino case, the RGE-triggered deviation of θ_{23} from 45° might be closely correlated with the neutrino mass ordering in the Majorana case.

In Eq. (4) it should be noted that the two Majorana CP-violating phases ρ and σ in the standard parametrization of U affect the running behaviors of Δ_i via $\cos \Phi_{ij}$ and $\sin \Phi_{ij}$. One should also note that $\mathcal{J} = (\sin 2\theta_{12} \sin 2\theta_{13} \cos \theta_{13} \sin 2\theta_{23} \sin \delta) / 8$, which only depends on the Dirac CP-violating phase δ , measures the strength of CP violation in neutrino oscillations. Therefore, $\delta \sim 270^\circ$ is especially favorable for significant CP-violating effects in the lepton sector, no matter whether the massive neutrinos are Dirac or Majorana particles.

3 Numerical results for Δ_i , θ_{23} and δ

We proceed to numerically illustrate the effects of μ - τ symmetry breaking regarding the PMNS matrix U — namely, the quantities Δ_i run from $\Delta_i = 0$ (i.e., $\delta_{23} = 45^\circ$ and $\delta = 270^\circ$) at $\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to $\Lambda_{\text{EW}} \sim 10^2$ GeV via the one-loop RGEs obtained in Eq. (3) or (4). Given a proper value of $\tan \beta$, the values of m_1 , Δm_{21}^2 , Δm_{31}^2 , θ_{12} and θ_{13} at $\Lambda_{\mu\tau}$ should be carefully chosen such that the best-fit results of Δm_{21}^2 , Δm_{31}^2 , θ_{12} , θ_{13} , θ_{23} and δ at Λ_{EW} as listed in Table 1 can all be reproduced to a good degree of accuracy³. If this strategy is workable, then the deviation of θ_{23} from 45° and that of δ from 270° will be purely attributed to the RGE-triggered PMNS μ - τ symmetry breaking effects⁴.

Table 1: The best-fit values of Δm_{21}^2 , Δm_{31}^2 , θ_{12} , θ_{13} , θ_{23} and δ obtained from two recent global analyses of current neutrino oscillation data [3, 4].

Reference	Mass ordering	Δm_{21}^2 (eV ²)	Δm_{31}^2 (eV ²)	θ_{12}	θ_{13}	θ_{23}	δ
Capozzi <i>et al</i> [3]	Normal	7.54×10^{-5}	$+2.47 \times 10^{-3}$	33.7°	8.8°	41.4°	250°
	Inverted		-2.34×10^{-3}		8.9°	42.4°	236°
Forero <i>et al</i> [4]	Normal	7.60×10^{-5}	$+2.48 \times 10^{-3}$	34.6°	8.8°	48.9°	241°
	Inverted		-2.38×10^{-3}		8.9°	49.2°	266°

3.1 Dirac neutrinos

For simplicity, we fix $\tan \beta = 31$ and input $m_1 = 0.1$ eV at $\Lambda_{\mu\tau}$, where Δ_1 , Δ_2 and Δ_3 are vanishing (or equivalently, $\theta_{23} = 45^\circ$ and $\delta = 90^\circ$), in our numerical calculations. Table 2 shows the input and output values of all the relevant parameters for two examples, which are based on the best-fit results reported by Capozzi *et al* [3] and Forero *et al* [4], respectively. Figure 1 illustrates how Δ_i evolve in either example. Some comments and discussions are in order.

(1) Given the inverted neutrino mass ordering, the best-fit results of the six neutrino oscillation parameters Δm_{21}^2 , Δm_{31}^2 , θ_{12} , θ_{13} , θ_{23} and δ at Λ_{EW} in Example I [3] can successfully be reproduced from the proper inputs at $\Lambda_{\mu\tau}$. In this case $\theta_{23}(\Lambda_{\text{EW}})$ lies in the first octant, and $\theta_{23}(\Lambda_{\mu\tau}) - \theta_{23}(\Lambda_{\text{EW}}) \simeq 2.6^\circ$ holds thanks to the RGE running effect. At the same time, we obtain $\delta(\Lambda_{\mu\tau}) - \delta(\Lambda_{\text{EW}}) \simeq 34^\circ$. Hence the RGE evolution can also provide a resolution to the quadrant of δ .

³Note that the notations $\delta m^2 \equiv m_2^2 - m_1^2$ and $\Delta m^2 \equiv m_3^2 - (m_1^2 + m_2^2)/2$ have been used in Ref. [3]. They are related with Δm_{21}^2 and Δm_{31}^2 as follows: $\Delta m_{21}^2 = \delta m^2$ and $\Delta m_{31}^2 = \Delta m^2 + \delta m^2/2$.

⁴Note that the ν fit group's best-fit results [16] are not taken into account in our numerical examples, because they happen to correspond to the disfavored cases listed in Table 1 (i.e., $\theta_{23} < 45^\circ$ for the normal neutrino mass ordering or $\theta_{23} > 45^\circ$ for the inverted ordering, in conflict with our expectations shown in Tables 2 and 3, respectively).

Table 2: The RGE-triggered PMNS μ - τ symmetry breaking effects for Dirac neutrinos running from $\Delta_i = 0$ at $\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to $\Lambda_{EW} \sim 10^2$ GeV in the MSSM with $\tan\beta = 31$.

Parameter	Example I (Capozzi <i>et al</i> [3])		Example II (Forero <i>et al</i> [4])	
	Input ($\Lambda_{\mu\tau}$)	Output (Λ_{EW})	Input ($\Lambda_{\mu\tau}$)	Output (Λ_{EW})
m_1 (eV)	0.100	0.093	0.100	0.093
Δm_{21}^2 (eV ²)	1.82×10^{-4}	7.54×10^{-5}	1.96×10^{-4}	7.60×10^{-5}
Δm_{31}^2 (eV ²)	-2.60×10^{-3}	-2.34×10^{-3}	3.00×10^{-3}	2.48×10^{-3}
θ_{12}	10.8°	33.6°	10.3°	34.6°
θ_{13}	9.4°	8.9°	8.4°	8.8°
θ_{23}	45.0°	42.4°	45.0°	48.4°
δ	270°	236°	270°	237°
\mathcal{J}	-0.015	-0.029	-0.013	-0.029
Δ_1	0	0.053	0	0.114
Δ_2	0	-0.141	0	0.001
Δ_3	0	0.088	0	-0.115

(2) In contrast, only the normal neutrino mass ordering allows us to obtain $\theta_{23}(\Lambda_{EW}) \simeq 48.4^\circ$ from $\theta_{23}(\Lambda_{\mu\tau}) = 45^\circ$ via the RGE evolution as shown in Example II [4]. Moreover, we obtain $\delta(\Lambda_{EW}) \simeq 237^\circ$ from $\delta(\Lambda_{\mu\tau}) = 270^\circ$, and this result is also consistent very well with the corresponding best-fit value $\delta \simeq 241^\circ$ as listed in Table 1. The future experimental data will only verify one of the above two possibilities for the octant of θ_{23} , but it will be interesting to test the expected correlation between the neutrino mass ordering and the deviation of θ_{23} (or δ) from 45° (or 270°).

(3) Figure 1 shows the behaviors of three PMNS μ - τ asymmetries Δ_i evolving from $\Lambda_{\mu\tau}$ down to Λ_{EW} for the two examples under discussion. In view of $\Delta_3 = \cos^2 \theta_{13} \cos 2\theta_{23}$ in Eq. (2), one must have $\Delta_3(\Lambda_{EW}) > 0$ for $\theta_{23}(\Lambda_{EW}) < 45^\circ$ in Example I, and $\Delta_3(\Lambda_{EW}) < 0$ for $\theta_{23}(\Lambda_{EW}) > 45^\circ$ in Example II. In comparison, the running behaviors of Δ_1 and Δ_2 are not so straightforward, because they depend on all the three flavor mixing angles and the CP-violating phase δ . But $\Delta_1 + \Delta_2 + \Delta_3 = 0$ holds at any energy scale between Λ_{EW} and $\Lambda_{\mu\tau}$, as one can see in Figure 1.

3.2 Majorana neutrinos

In this case we simply fix $\tan\beta = 30$ and input $m_1 = 0.1$ eV at $\Lambda_{\mu\tau}$, where $\Delta_1 = \Delta_2 = \Delta_3 = 0$ holds, in our numerical calculations. Table 3 is a brief summary of the input and output values of all the relevant parameters for Example I [3] and Example II [4], respectively. In addition, Figure 2 illustrates how the three PMNS μ - τ asymmetries evolve from $\Lambda_{\mu\tau}$ down to Λ_{EW} in either example.

Although the present case involves two extra CP-violating phases ρ and σ , the running behaviors of Δ_i in Figure 2 are quite similar to those in Figure 1. Of course, one has to adjust the initial values of ρ and σ at $\Lambda_{\mu\tau}$ in a careful way, such that the best-fit results of the six neutrino oscillation parameters can correctly be reproduced at Λ_{EW} . We find that it is really possible to resolve the octant of θ_{23} and the quadrant of δ at the same time via radiative PMNS μ - τ symmetry breaking. Very similar to the Dirac neutrino case, the RGE-triggered deviation of θ_{23} from 45° in the Majorana case is also closely correlated with the neutrino mass ordering. Namely, $\theta_{23} \simeq 42.4^\circ$ reported in

Ref. [3] (or $\theta_{23} \simeq 48.9^\circ$ reported in Ref. [4]) at Λ_{EW} can evolve from $\theta_{23} = 45^\circ$ at $\Lambda_{\mu\tau}$ only when the neutrino masses have an inverted (or normal) ordering.

Table 3: The RGE-triggered PMNS μ - τ symmetry breaking effects for Majorana neutrinos running from $\Delta_i = 0$ at $\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to $\Lambda_{\text{EW}} \sim 10^2$ GeV in the MSSM with $\tan\beta = 30$.

Parameter	Example I (Capozzi <i>et al</i> [3])		Example II (Forero <i>et al</i> [4])	
	Input ($\Lambda_{\mu\tau}$)	Output (Λ_{EW})	Input ($\Lambda_{\mu\tau}$)	Output (Λ_{EW})
m_1 (eV)	0.100	0.087	0.100	0.087
Δm_{21}^2 (eV ²)	1.70×10^{-4}	7.54×10^{-5}	2.12×10^{-4}	7.60×10^{-5}
Δm_{31}^2 (eV ²)	-2.98×10^{-3}	-2.34×10^{-3}	3.50×10^{-3}	2.48×10^{-3}
θ_{12}	35.2°	33.7°	32.1°	34.6°
θ_{13}	10.1°	8.9°	6.9°	8.8°
θ_{23}	45.0°	42.4°	45.0°	48.9°
δ	270°	236°	270°	241°
ρ	-82°	-66°	-76°	-45°
σ	19°	27°	17°	29°
\mathcal{J}	-0.040	-0.029	-0.027	-0.030
Δ_1	0	0.054	0	0.111
Δ_2	0	-0.142	0	0.022
Δ_3	0	0.088	0	-0.133

In view of the fact that the present best-fit results of θ_{23} and δ are still quite preliminary, we foresee that they must undergo some changes before they are well determined by the more precise experimental data in the near future. Hence our numerical analysis is not targeted for a complete parameter-space exploration but mainly for the purpose of illustration [17]. Its outcome supports our original conjecture: the slight μ - τ symmetry breaking behind the observed pattern of lepton flavor mixing can originate from the RGE evolution from a superhigh flavor symmetry scale down to the electroweak scale. Note that there are two adjustable unknown parameters in our calculations: the absolute neutrino mass m_1 and the MSSM parameter $\tan\beta$. Once m_1 is experimentally determined and $\tan\beta$ is theoretically fixed, for example, it will be interesting to see whether one can still resolve the octant of θ_{23} and the quadrant of δ with the help of radiative PMNS μ - τ symmetry breaking effects.

We admit that the present best-fit result $\delta \sim 270^\circ$ remains too preliminary. In fact, there is not any nontrivial region associated with the allowed values of δ at the 2σ level [3, 4]. Hence it also makes sense to look at the RGE-triggered corrections to $\theta_{23} = 45^\circ$ and $\delta = 90^\circ$ for the energy scale to evolve from $\Lambda_{\mu\tau}$ down to Λ_{EW} . This possibility has already been discussed in some literature (see, e.g., Refs. [17, 18]). Once the CP-violating phase δ is measured or constrained to a better degree of accuracy in the near future, it will be possible to examine whether the quantum corrections can really accommodate the observed effect of PMNS μ - τ symmetry breaking or not.

4 Summary and further discussions

To summarize, we have conjectured that the PMNS μ - τ permutation symmetry is exact at a superhigh energy scale $\Lambda_{\mu\tau} \sim 10^{14}$ GeV, where the origin of neutrino masses and flavor mixing has a good dynamic reason, and its slight breaking happens via the RGE running down to the electroweak scale $\Lambda_{EW} \sim 10^2$ GeV. This idea is particularly interesting in the sense that it can help resolve the octant of θ_{23} and the quadrant of δ at the same time thanks to radiative PMNS μ - τ symmetry breaking in the MSSM. In fitting current neutrino oscillation data we have found that the RGE-triggered deviation of θ_{23} from 45° is uniquely correlated with the neutrino mass ordering: $\theta_{23} \simeq 42.4^\circ$ [3] (or $\theta_{23} \simeq 48.9^\circ$ [4]) at Λ_{EW} can naturally originate from $\theta_{23} = 45^\circ$ at $\Lambda_{\mu\tau}$ if the neutrino mass ordering is inverted (or normal). Accordingly, the preliminary best-fit results of δ at Λ_{EW} can also evolve from $\delta = 270^\circ$ at $\Lambda_{\mu\tau}$. Such remarkable findings are independent of any specific models of neutrino mass generation and lepton flavor mixing, and they will soon be tested in the upcoming neutrino oscillation experiments.

Note that some previous studies of the RGE evolution of lepton flavor mixing parameters have more or less involved the μ - τ symmetry breaking effects [19]. In this connection a few constant neutrino mixing patterns which possess $|U_{\mu i}| = |U_{\tau i}|$, such as the bimaximal [20] and tri-bimaximal [21] ones with $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$, have been assumed at a superhigh energy scale; and their RGE running behaviors have been investigated mainly to see whether a finite θ_{13} can be radiatively generated at low energies [18]. The closest example of this kind should be the work [22] on radiative corrections to the tetra-maximal neutrino mixing pattern [23], in which $\theta_{13} \simeq 8.4^\circ$, $\theta_{23} = 45^\circ$ and $\delta = 90^\circ$ or 270° have been predicted. Our present work is different from the previous ones in several aspects: (a) it is not subject to any explicit neutrino mixing pattern; (b) it focuses on the PMNS μ - τ asymmetries Δ_i and its RGE evolution; (c) it provides a reasonable resolution to the octant of θ_{23} by attributing it to the PMNS μ - τ symmetry breaking effect; (d) it may also resolve the quadrant of δ in a similar way. All in all, we have established the RGE connection between a given neutrino mass model with the exact μ - τ symmetry at superhigh energies and the neutrino oscillation parameters at low energies. Such a connection is expected to be very useful for neutrino phenomenology in the era of precision measurements.

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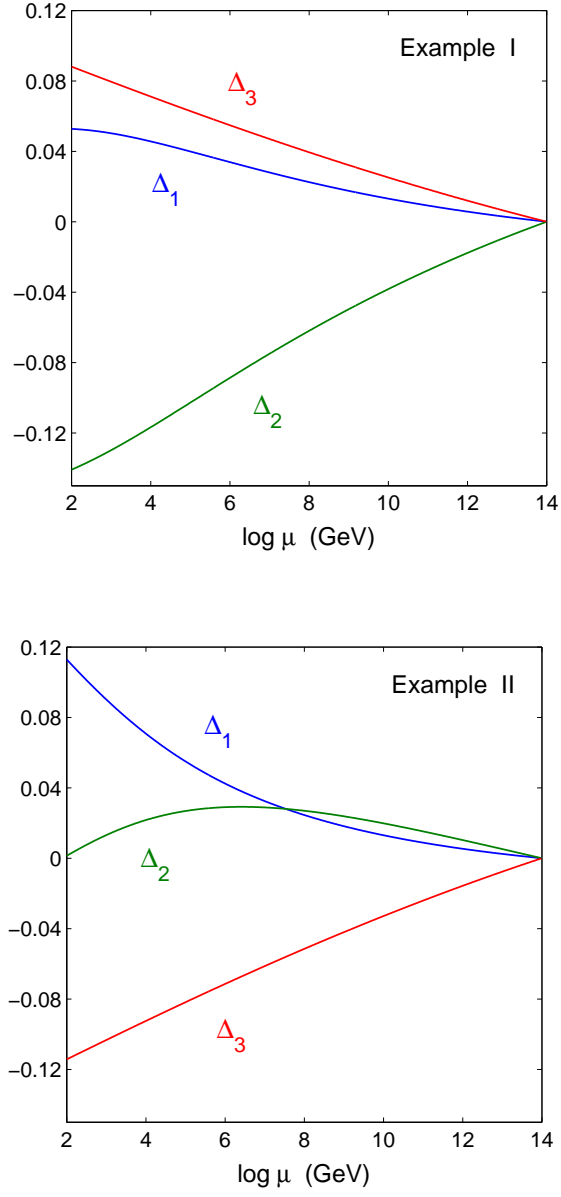


Figure 1: The RGE-triggered μ - τ symmetry breaking effects for Dirac neutrinos running from $\Delta_i = 0$ at $\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to $\Lambda_{EW} \sim 10^2$ GeV in the MSSM with $\tan \beta = 31$.

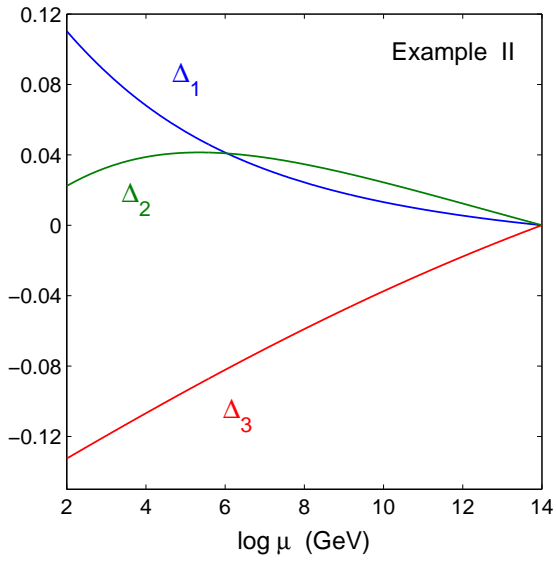
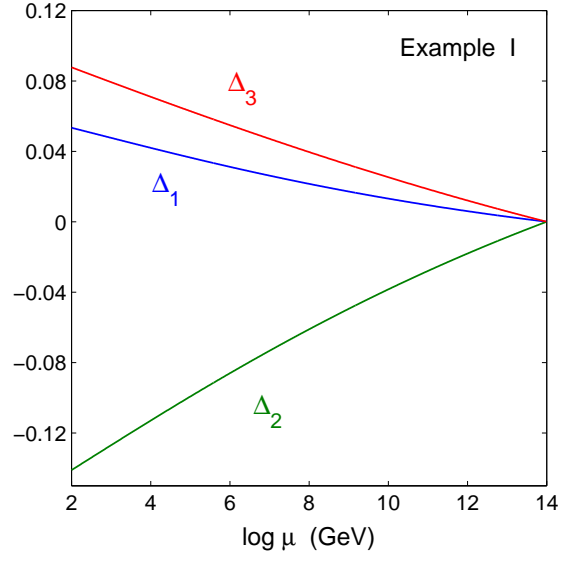


Figure 2: The RGE-triggered μ - τ symmetry breaking effects for Majorana neutrinos running from $\Delta_i = 0$ at $\Lambda_{\mu\tau} \sim 10^{14}$ GeV down to $\Lambda_{EW} \sim 10^2$ GeV in the MSSM with $\tan \beta = 30$.